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RIVER CHANNEL ROUGHNESS

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Assoc. Members, ASCE

HYDRAULICS DIVISION

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PAPERS

RIVER CHANNEL ROUGHNESS

By Hans A. Einstein[1] and Nicholas L. Barbarossa,[2]
Assoc. Members ASCE

Synopsis

Solution of the problem of determining the frictional losses in natural streams usually depends on the judgment and experience of the engineer. A rational approach to this problem is offered in this paper. Exisitng theory on friction in open channels is reviewed, and a logical derivation leads to the development of formulas and charts to determine the relationship between stage and discharge in a given reach of river. An interdependent treatment of the hydraulic characteristics of flow and the sedimentary characteristics of the stream is presented. Friction losses due to grain roughness, channel irregularities, and other factors are studied and formulated, and suggestions are made for practical application of a proposed rational method of friction evaluation.

Introduction

In dealing with natural streams with movable beds, the engineer is confronted with the problem of evaluating frictional losses. Unless adequate and complete data are available to guide him, too often not the case, his evaluation of frictional effects is based almost entirely upon judgment. The element of doubt inevitably present forces the engineer to be conservative. The degree of conservatism exercised is reflected in project costs. If the element of doubt and confusion concerning frictional losses in natural streams could be reduced to the lowest practicable limit, substantial savings in the costs of many hydraulic projects undoubtedly would result.

Although many data concerning the roughness values of natural streams have been accumulated and published, the fact remains that, in estimating the roughness value of a particular stream for which no roughness data are available, there are likely to be as many different estimates as there are estimators. Even when

Note.—Written comments are invited for publication; the last discussion should be submitted by January, 1951.

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roughness data are available for a particular stream, often these data are applicable only within a very narrow range of discharge and for a particular location on the stream. It is not uncommon for many engineers to think of a natural river channel as having a single roughness value for all discharges. Even some of those engineers who feel that the roughness value n (as applied to Manning's or Kutter's formula) varies with the discharge Q are compelled, by lack of data concerning the variation of n with Q, to assign a single-valued n to a particular stream.

It is evident that a new and more rational approach to the evaluation of frictional losses in natural channels is needed. The writers believe that a realistic appraisal of frictional losses in streams with movable beds is possible only when the sedimentary characteristics of the stream and the usual hydraulic characteristics of flow are treated interdependently. With regard to the sedimentary characteristics, frictional losses in natural streams depend on the size and gradation of sediments composing the stream bed surface and on the shape, size, and spacing of irregularities within the boundaries of flow. It is because of the lack of these irregularities that laboratory roughness values derived under hydrodynamically similar conditions often do not agree with the roughness values of natural streams. It is probable that these irregularities are a function of the sediment transport which is certainly a function of flow conditions. The interdependence of the sedimentary and hydraulic conditions of flow is obvious. If the functional relationship between the irregularities of the bed and the sediment transport can be expressed in dimensionless form and evaluated by means of experimental data, it will be possible then for the practicing engineer to use this relationship as a means of evaluating the frictional losses of natural streams with considerably more confidence than in the past.

Existing Theory

Turbulence theories have substantiated beyond doubt that the distribution of flow velocities in a real fluid (with internal friction) is very closely related to the shear stresses in the fluid. Since the flow of water in an open stream channel is influenced by friction along its solid boundaries, the bed and the banks, the frictional effect on the flow may be measured by the shear stresses along these boundaries.

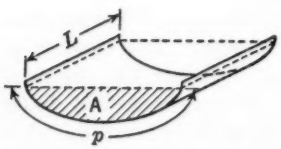

Fig. 1.—Cross Section of Flow

In Fig. 1 the total shear force along the wetted boundary surface $L\ p$ is $L\ p\ \tau_o$ in which τ_o is defined as the average unit shear stress, parallel to the direction of flow, within the body of water of length L and wetted perimeter p. This body of water, of cross-sectional area A, has a weight component in the direction of flow of $A\ L\ w\ S$, in which w is the unit weight of water and the slope S of the water surface is small. For the case of uniform motion the algebraic sum of the forces acting parallel to the direction of flow must equal zero since no acceleration of flow takes place. Neglecting air friction at the open surface and noting that the total shear force acts in a retarding direction, it follows that

$$A\ L\ w\ S - L\ p\ \tau_o = 0 \qquad (1)$$

If $\rho = w/g$ is the unit mass density, in which g is the acceleration due to gravity, Eq. 1 may be rewritten as

$$\tau_o = \frac{w\,S\,A}{p} \qquad (2a)$$

and

$$\frac{\tau_o}{\rho} = \frac{g\,S\,A}{p} \qquad (2b)$$

Both forms of Eq. 2 are equally valuable. The dimensions of the channel appear only in the ratio $A/p = R$, commonly known as the hydraulic radius. In the derivations of this paper it will prove most helpful, however, to think of the hydraulic radius as the ratio $\frac{A\,L}{p\,L}$, the ratio of the volume of the flow segment to the corresponding frictional surface area.

Fluid mechanics teaches that on a hydrodynamically smooth wall the shear forces are transmitted entirely by the tangential shearing stresses within the fluid, whereas on a hydrodynamically rough wall part of the shear forces are transmitted by unsymmetrically distributed normal stresses on the individual roughness protrusions of the wall. In the latter case the degree of resistance to flow depends to a great extent upon the size, shape, and spacing of the individual irregularities. This type of resistance is commonly called form resistance. The actual flow pattern in the immediate vicinity of a rough wall is basically different from that near a smooth wall. However, at a distance away from the wall of one or two roughness lengths K_s this difference disappears entirely, and the two flow patterns become identical.

Roughness of a Natural Stream Bed

Derivations.—The rough wall discussed so far is a wall with a surface characterized by straight alinement and practically uniform roughness. The pipes which J. Nikuradse[3,4] roughened by gluing sand grains to the inside are a good example of such a straight, uniformly rough surface. The individual grains of bed sediment of a natural stream also provide a grain roughness which is approximately comparable to that of Mr. Nikuradse's roughened pipes, and the resulting frictional losses due to the grain roughness alone may be calculated in much the same way as are the frictional losses for pipes or artificial uniform channels of immovable beds. However, the natural stream bed surface, especially that of a sediment-carrying alluvial stream, is basically different from that of Mr. Nikuradse's experimental pipes because the surface is not straight. It is often irregular due to sediment bars of various scales.

The foremost problem confronting engineers who deal with natural streams is to evaluate the influence of the irregularities of the stream bed surface upon the grain roughness. It is very likely that these surface irregularities are a source of additional friction losses over and above those caused by the grain roughness alone. The question is can these additional friction losses be described physically. A clue to the answer may be found from a consideration of

[3] "Gesetzmässigkeiten der turbulenten Strömung in glatten Rohren," by J. Nikuradse, *Forschungsheft 356*, VDI-Verlag, Berlin, 1932.
[4] "Strömungsgesetze in rauhen Rohren," by J. Nikuradse, *Forschungsheft 361*, VDI-Verlag, Berlin, 1933.

the problem involving the evaluation of friction losses in a rough pipe line which has irregularities such as elbows, enlargements, contractions, or any other type of transitional fittings. It is common knowledge that, despite the very serious theoretical objections due to the basic nonlinearity of the problem, one may with a reasonable degree of accuracy determine individually the friction losses in each element of the pipe line and simply add them to obtain the total loss. By neglecting the mutual interference of flow patterns within the different elements of the pipe line, the problem is thus reduced to that of calculating the individual losses. The same type of approach is suggested in this paper for the determination of the additional friction loss due to irregularities of natural streams with moving sediment beds.

The friction loss due to grain roughness may be evaluated by use of the Manning equation or by any other suitable open channel formula. In the Manning equation with the average velocity

$$V = \frac{1.486}{n} R^{\frac{2}{3}} S^{\frac{1}{2}} \quad \ldots\ldots (3)$$

the roughness factor n may be replaced by the grain roughness diameter K_s by means of the empirical formula[5] derived by K. Strickler:

$$n = \frac{\sqrt[6]{K_s}}{29.3} \quad \ldots\ldots (4)$$

in which K_s is measured in feet. By means of this substitution and by use of Eqs. 2, Eq. 3 may be rearranged into the form:

$$\frac{V}{S^{\frac{1}{2}} R^{\frac{1}{2}} g^{\frac{1}{2}}} = \frac{V}{\sqrt{\tau_0/\rho}} = 7.66 \left(\frac{R}{K_s}\right)^{1/6} \quad \ldots\ldots (5)$$

It follows from the way Eqs. 2 were derived that in Eq. 5 R is the hydraulic radius pertaining to the grain roughness. Stated another way R is the ratio of the cross-sectional area A (contributing the energy that is converted into turbulence along the perimeter p of representative grain roughness K_s) and the perimeter p. Call this area A' and the corresponding hydraulic radius R'. If the abbreviation V'_f is introduced for $\sqrt{S R' g} = \sqrt{\tau'_0/\rho}$, Eq. 5 becomes

$$\frac{V}{V'_f} = 7.66 \left(\frac{R'}{K_s}\right)^{1/6} \quad \ldots\ldots (6)$$

This is the Manning-Strickler equation. It may be replaced by a logarithmic equation that is better supported by theory:

$$\frac{V}{V'_f} = 5.75 \log_{10} \left(12.2 \frac{R'}{K_s}\right) \quad \ldots\ldots (7)$$

This equation gives for a very wide range of conditions ($4 < R'/K_s < 2{,}000$) practically the same results as Eq. 6.[6] Eq. 7 seems to be very reliable in de-

[5] "Beiträge zur Frage der Geschwindigkeitsformel und der Rauhigkeitszahlen für Strom Kanäle und geschlossene Leitungen," by K. Strickler, Mitt. No. 16 des Eidgenössische Amtes für Wasser wirtschaft, Bern, Switzerland, 1923.

[6] "Flow on a Movable Bed," by H. A. Einstein, *Proceedings of the Second Hydraulic Conference*, Iowa Inst. of Hydr. Research, State Univ. of Iowa, Iowa City, Iowa, 1942, p. 332.

scribing all flows on hydrodynamically rough surfaces. For such flows the ratio of the roughness diameter K_s to the theoretical thickness δ' of the laminar sublayer is[7]

$$\frac{K_s}{\delta'} = \frac{K_s V'_f}{11.6\,\nu} > 5 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

in which ν is the kinematic viscosity.

If K_s/δ' is smaller than 5, Eq. 7 must be corrected. The correction may be made by introducing a factor x into the equation which then becomes

$$\frac{V}{V'_f} = 5.75 \log_{10}\left(12.2\,\frac{R'}{K_s}\,x\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

in which the correction x is a function of K_s/δ' and is given by Fig. 2.[8] This

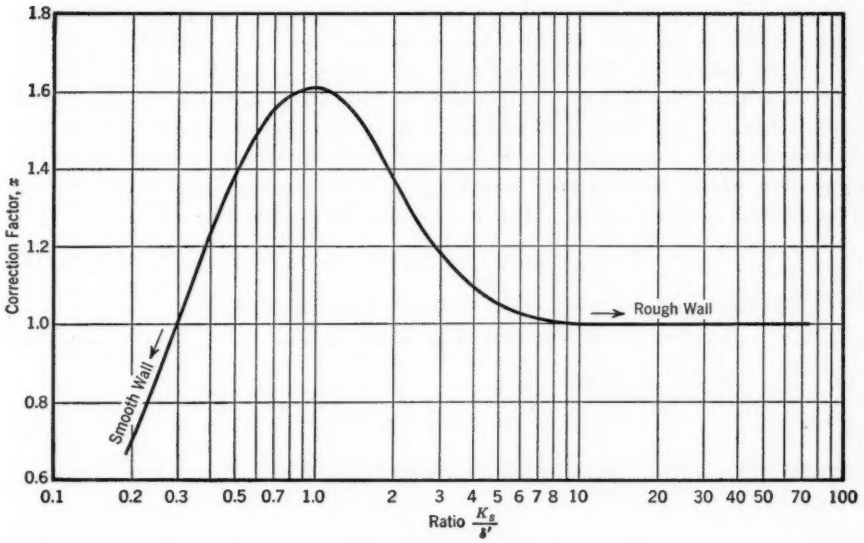

FIG. 2.—CORRECTION X IN THE LOGARITHMIC FRICTION FORMULA

relationship is derived from Nikuradse's measurements,[4] of flows over sand surfaces. (In some practical applications the factor x may be neglected since its effect upon the ratio V/V'_f can never be more than about 20 percent.) Besides these usual friction losses attributable to the grain roughness in the natural stream, there are additional friction losses whose origin is connected in some manner with the presence of sand bars, sand waves, and other irregularities of the channel. For the easiest understanding of how these irregularities generate additional friction losses, imagine them to be uniformly sized and uniformly spaced protrusions of the stream bed, such as piles or single rocks. Assuming N to be the number of protrusions on the surface area $L\,p$ (Fig. 1),

[7] "Fluid Mechanics for Hydraulic Engineers," by Hunter Rouse, *Engineering Societies Monographs*, McGraw-Hill Book Co., Inc., New York, N. Y., 1938, p. 243, Eq. 171.

[8] *Ibid.*, Fig. 123.

each protrusion having a cross-sectional area a diverting the flow, the drag force f on each protrusion may be expressed as:

$$f = c_D\, a\, w \frac{V^2}{2g} \quad \ldots\ldots (10)$$

in which c_D is the drag coefficient of the protrusion, depending on its shape and size. The total drag force, in the direction of flow, on N protrusions is then

$$F = N\, c_D\, a\, w \frac{V^2}{2g} \quad \ldots\ldots (11)$$

The additional shear stress τ''_o on the boundary surface $L\,p$ due to this total drag force is

$$\tau''_o = \frac{F}{L\,p} = \frac{N\,a}{L\,p} c_D\, w \frac{V^2}{2g} \quad \ldots\ldots (12)$$

The total shear force $L\,p\,\tau_o$ of Eq. 1 may be considered to include two components: the shear force due to the grain roughness $L\,p\,\tau'_o$ and the shear force due to the irregularities $L\,p\,\tau''_o$. Similarly, the total area A of the cross section in Fig. 1, contributing energy that is converted to turbulence at the bed, may be broken down into two parts A' and A''. A' is that part of the total area which pertains to the grain roughness as previously defined, and A'' is that part of the total area which contributes the energy that is spent in friction due to the irregularities of the channel. Eq. 1 may now be written as

$$(A' + A'')\, L\, w\, S - L\, p\, (\tau'_o + \tau''_o) = 0 \quad \ldots\ldots (13)$$

Dividing Eq. 13 by $L\,p$ and designating R'' as A''/p gives

$$w\, S\, R' + w\, S\, R'' = \tau'_o + \tau''_o \quad \ldots\ldots (14)$$

Since $\tau'_o = w\, S\, R'$, it follows that

$$\tau''_o = w\, S\, R'' \quad \ldots\ldots (15)$$

Eliminating τ''_o from Eq. 12 and Eq. 15 and letting $V''_f = \sqrt{g\, S\, R''}$ then

$$\frac{V}{V''_f} = \sqrt{\frac{2\, L\, p}{N\, a\, c_D}} \quad \ldots\ldots (16)$$

It will be noted that the derivation of Eq. 16 depends on the basic assumption that the grain roughness and the irregularities are uniformly distributed over the boundary surface $L\,p$.

The entire expression on the right side of Eq. 16 is a function of the shape, size, and frequency of the irregularities along the surface area $L\,p$. Now, thinking of the irregularities (or protrusions) as sand bars, it is most natural to expect them to be a function of the sediment transport along the channel bed. This sediment transport is itself a function of the shear stress τ'_o on the sediment grains and of the representative grain diameter D. The effect of the variables τ'_o and D is reflected in the dimensionless variable[9]

$$\psi' = \frac{\rho_s - \rho_f}{\rho_f} \frac{D}{R'\, S} \quad \ldots\ldots (17)$$

[9] "Formulas for the Transportation of Bed Load," by H. A. Einstein, *Transactions*, ASCE, Vol. 107, 1942, p. 561.

in which ρ_s is the unit mass density of the sediment in air and ρ_f is the unit mass density of water. Therefore, V/V''_f is a function of ψ'. This relationship must be determined from actual river measurements.

Determination of Additional Friction Losses Due to Channel Irregularities.— These losses are reflected in the friction velocity ($V''_f = \sqrt{g\,S\,R''}$) of Eq. 16. In an absolutely uniform channel these losses are obviously equal to zero, and, therefore, V''_f is never zero in a real channel but varies, depending upon the degree of the irregularities. The effect of these irregularities is to distort the flow pattern. When the discharge is least, the distortion of the flow pattern is greatest (as witness the meandering of natural stream at low flows). As the

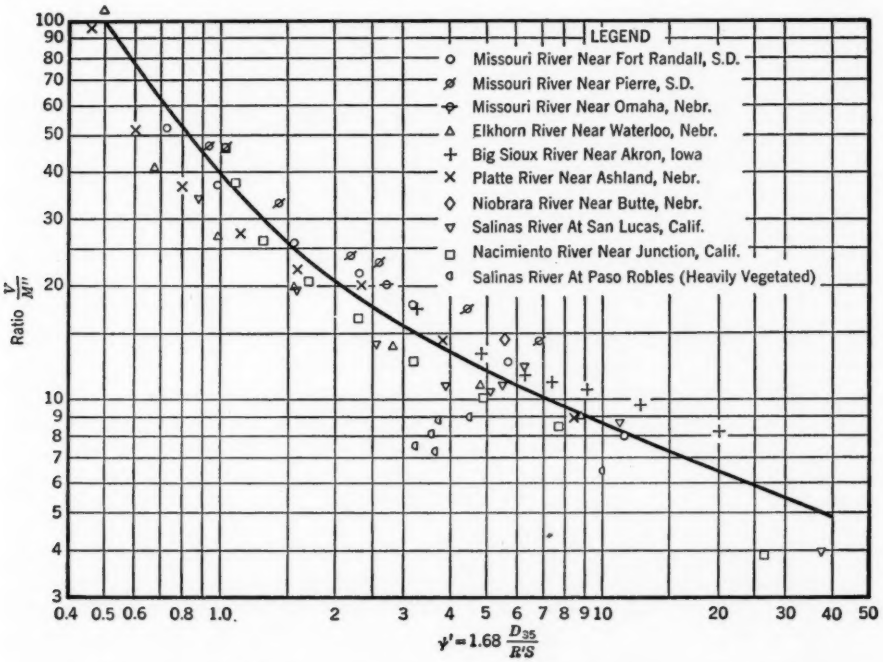

FIG. 3.—FRICTION LOSS DUE TO CHANNEL IRREGULARITIES AS A FUNCTION OF SEDIMENT TRANSPORT

discharge increases (and hence the sediment transport along the bed also increases) the distortion of the flow pattern becomes less and less because the alinement of flow becomes progressively straighter. Consequently, one may expect that the additional friction loss (or V''_f) diminishes as the discharge Q increases. Since both V and R' become larger with increasing Q, it follows that the ratio V/V''_f also increases, but the ratio ψ' from Eq. 17 decreases. (Note that an increasing transport of sediment means that ψ' decreases.) The relationship between V/V''_f and ψ' has been determined from actual measurements for a number of rivers, most of which are in the Missouri River Basin. The results obtained are indicated in Fig. 3. Except for heavily vegetated streams, there appears to be a fairly well-defined relationship between V/V''_f and ψ'. A tentative curve for the relationship sought has been drawn on Fig. 3.

The supporting computation for the results shown in Fig. 3 are based on the following procedure:

1. A comparatively long reach of river was selected within which there exists a gaging station where the relationship between discharge and water surface elevation is known;

2. The over-all slope of the reach was determined as the total drop divided by total distance;

3. Surveyed cross sections of the river channel spaced more or less uniformly throughout the reach were then superimposed upon one another at the gaging station, using the over-all slope as a means of bringing the cross sections together to a common water surface line at the gaging station;

4. The average river width for this common water surface line (correlated with discharge at the gaging station) was obtained, and then the average widths at depth intervals referred to the common line were calculated. Thus, the average cross section for the entire reach of river was determined.

5. Mechanical analyis curves were constructed for all bed surface samples (at least 3 or 4 samples at each cross section obtained by drag bucket in the active channel) and an average, composite mechanical analysis curve was then determined by taking the average of the percentage values from each individual curve for each chosen grain size.

6. The value of K_s to be used in Eqs. 6, 7, 8, and 9 represents the grain roughness. The grain size on the average mechanical analysis curve at which 65% is finer was taken as a representative value of K_s.[9]

7. Hydraulic computations were made using the average cross section, the over-all slope S, and the stage-discharge relationship at the gaging station. Thus, for every chosen discharge the cross-sectional characteristics A and R of the average section were known. The hydraulic computations are based upon the following assumptions:

a. The water surface profile is essentially a straight line in a long reach and remains straight with unvarying slope as the discharge changes.

b. The average cross section in a fairly long reach does not substantially change in shape and size as the discharge varies. (There is some supporting evidence for this assumption.)

c. Because of *a* and *b* the uniform flow equations may be used with negligible error.

8. Eqs. 7, 8, and 9 were then used to determine R', with all other quantities known in these equations.

9. The total R was determined as A/p, and the R'' was determined as $R - R'$. The latter step is permissible because both R' and R'' are derived from the same wetted perimeter p, and A equals $A' + A''$ as defined.

10. With R' and R'' known, the dimensionless values of V/V''_* and ψ' were finally evaluated and plotted on Fig. 3. By definition $V''_* = \sqrt{g\,S\,R''}$ and $\psi' = 1.68\,D/R'S$ from Eq. 17. The ratio $\frac{\rho_s - \rho_f}{\rho_f}$ which represents the density of the bed sediment under water with respect to the density of water may be

assumed to have a value between 1.65 and 1.70 for most natural sediment mixtures, with 1.68 a fair average. The representative grain diameter D of the sediment that is transported along the stream bed was chosen as that grain size on the average mechanical analysis curve that is 35% finer.[9]

11. In making computations for comparatively narrow streams with steep banks, the friction loss on these banks was calculated separately as friction on a rough wall, using Manning's formula (with V and S known and n assigned) to determine the hydraulic radius R_w pertaining to these banks. Then the area

FIG. 4.—TYPICAL RIVER REACHES STUDIED

$A_w = R_w \times$ wetted perimeter of the banks was deducted from the total cross-sectional area A to give $A' + A''$. With $A' = R' \times$ total wetted perimeter less wetted perimeter of the banks, A'' was determined as $A - A' - A_w$. R'' was then computed as the quotient of A'' divided by the same wetted perimeter that was used to evaluate A'.

Description of River Reaches.—A brief description of the river reaches that were selected for study and pertinent data are given in Table I. The data furnished in Table I are complete enough to permit a computational check of at

TABLE 1.—SUMMARY OF DATA

River	Test Reach: Approximate location	Test Reach: Length (miles)	Test Reach: No. of sections	Test Reach: Water surface slope ($\times 10^4$)	Discharge (cu ft per sec): Minimum	Discharge (cu ft per sec): Maximum	Average Velocity (ft per sec): Minimum	Average Velocity (ft per sec): Maximum	Water Surface Width (T) (feet): Minimum	Water Surface Width (T) (feet): Maximum
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Missouri	Pierre, S. D.[b]	15.2	28	1.69	5,200	145,000	1.64	5.50	970	3,000
Missouri	Ft. Randall, S. D.[c]	35.5	23	1.49	5,200	145,000	1.11	6.12	1,038	2,130
Missouri	Omaha, Nebr.[d]	10.9	14	1.67	30,800	30,800	3.74	3.74	839	839
Elkhorn	Waterloo, Nebr.[e]	9.0	8	5.28	450	14,800	1.32	7.67	225	291
Big Sioux	Akron, Ia.[f]	29.5	5	1.89	550	8,550	1.14	3.46	132	184
Niobrara	Butte, Nebr.[g]	1.0	5	12.57	1,045	1,045	1.95	1.95	804	804
Platte	Ashland, Nebr.[h]	23.0	13	7.38	2,000	78,000	1.64	9.14	725	1,541
Salinas	San Lucas, Calif.[i]	7.0	7	11.60	55	43,500	0.61	6.67	130	2,050
Nacimiento	Junction, Calif.[j]	3.0	5	13.20	290	38,300	0.84	7.72	272	1,081
Salinas	Paso Robles, Calif.[k]	6.0	8	17.20	21	6,000	0.14	3.00	110	595

[a] Subscript to D refers to "percentage finer than" on average mechanical analysis curve. [b] Fairly straight many large sand bars. [d] River works for navigation have materially narrowed natural stream; sand bars not as correction made. [f] Winding stream; banks vegetated; no sand bars visible; bank correction made. [g] Very stream; many split channels; winding stream; great number of sand bars; range of stage for large change in Q is vegetation on banks. [k] Fairly straight channel divided by wood strips; heavily vegetated.

least one point for each stream indicated on Fig. 3, except the Elkhorn and Big Sioux Rivers for which bank corrections were made.

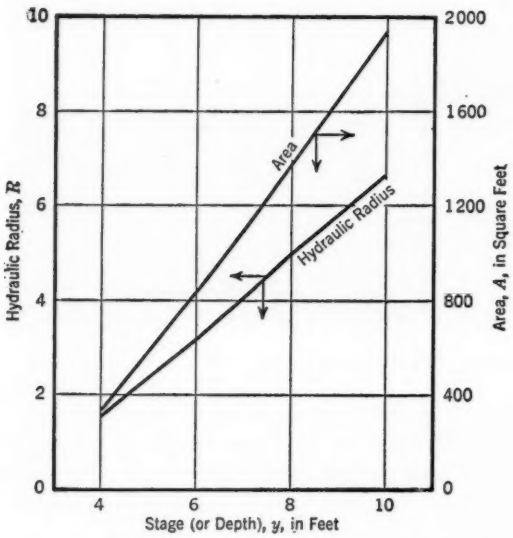

FIG. 5.—STAGE RELATIONSHIPS FOR PROBLEM STREAM

Practical Application of Relationship Between V/V''_f and ψ'.—When adequate hydraulic data are lacking for close determination of the stage-discharge relationship of a particular stream, the curve in Fig. 3 may be used to determine this relationship. How this may be done is best illustrated by an example. Suppose it is desired to determine the rating curve for a stream whose characteristics (having been determined by surveys and by application of procedures previously described) are as follows: (1) Slope $S = 5.00 \times 10^{-4}$; (2) cross-sectional dimensions of the average cross section of the channel in a fairly long reach as a function of stage y, as indicated in Fig. 5; (3) the grain size from the average mechanical analysis curve within the chosen reach at which 65% is finer is $K_s = 0.50$ mm, and the grain size from the same

ON RIVERS SELECTED FOR STUDY

PROPERTIES OF AVERAGE SECTION										SEDIMENT CHARACTERISTICS				
HYDRAULIC RADIUS (R) (FEET)		ROUGHNESS FACTOR (MANNING'S n)		RATIO T/R		R' (FEET)		δ' AT 60° F (FEET)		$K_s = D_{65}$ (feet)	D_{35} (feet)	RATIO K_s/δ'		Photo Fig. 4
Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum	Minimum	Maximum			Minimum	Maximum	
(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
3.26	8.79	0.015	0.026	297	341	0.880	6.73	0.00073	0.00201	0.000820	0.000590	0.41	1.12	....
4.53	11.13	0.014	0.045	191	229	0.520	8.28	0.00070	0.00280	0.000722	0.000525	0.26	1.03	(*a*)
9.83	9.83	0.024	0.024	85	85	3.420	3.42	0.00102	0.00102	0.001100	0.000918	1.08	1.08	....
1.52	6.62	0.016	0.034	44	148	0.240	4.26	0.00052	0.00218	0.000852	0.000657	0.39	1.63	(*b*)
3.64	13.40	0.033	0.042	14	36	0.460	2.87	0.00105	0.00263	0.001510	0.001050	0.57	1.44	(*d*)
0.67	0.67	0.021	0.021	1,200	1,200	0.210	0.21	0.00151	0.00151	0.000820	0.000602	0.54	0.54	....
1.68	5.54	0.014	0.034	278	432	0.280	5.16	0.00040	0.00171	0.001900	0.001050	1.11	4.75	(*c*)
0.69	3.18	0.016	0.065	189	644	0.050	2.14	0.00049	0.00320	0.002200	0.001300	0.69	4.50	(*e*)
1.27	4.59	0.019	0.076	214	236	0.150	3.95	0.00034	0.00170	0.025000	0.003300	15.00	73.00	(*f*)
0.45	3.36	0.046	0.262	177	244	0.015	0.55	0.00080	0.00480	0.006100	0.002300	1.30	7.60	(*g*)

reach; channel flow divided at places; numerous large sand bars. [c] Considerable winding of channel; split flow; prevalent. [e] Winding stream; banks vegetated; sand bars visible; probably slack water areas in channel; bank shallow stream; sinuous; great number of sand bars; split channels; vegetation negligible. [h] Very shallow, wide small. [f] Fairly straight channel; bed and banks all sand. [g] Fairly straight channel; bed sand and gravel;

curve at which 35% is finer is $D_{35} = 0.40$ mm. In feet, $K_s = 16.4 \times 10^{-4}$, and $D_{35} = 13.1 \times 10^{-4}$; (4) the bank friction is comparatively negligible, and the active channel (even though split by sand bars and possibly islands) is free of debris and vegetation.

The computations are carried out in tabular form after first determining the constants in Eqs. 6 and 17. For practical purposes Eq. 6 is more convenient to use than Eqs. 7 or 9 because solutions are direct. Using the data given above for the problem stream, Eq. 6 yields

$$V = \frac{7.66\sqrt{g\,S}}{(K_s)^{1/6}}(R')^{\frac{2}{3}} = \frac{7.66 \times 5.67 \times 2.24}{10^{4/3} \times 16.4^{1/6}} = (R')^{\frac{2}{3}} \quad \ldots\ldots (18a)$$

or

$$V = 2.83\,(R')^{\frac{2}{3}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18b)$$

From Eq. 17

$$\psi' = 1.68\,\frac{D_{35}}{R'\,S} = 1.68\,\frac{13.1 \times 10^{-4}}{5.00 \times 10^{-4}\,R'} = \frac{4.40}{R'} \quad \ldots\ldots\ldots (19)$$

The computations for three points on the desired rating curve are shown in Table 2. The procedure is: (*a*) assume R' such that Eqs. 18 will yield velocity values that reasonably may be expected for the given stream; (*b*) determine V and ψ' from Eqs. 18 and 19; (*c*) with the known value of ψ' read off the value of V/V''_f from the curve in Fig. 3; (*d*) then calculate V''_f and R''; (*e*) R is now determined as $R' + R''$, and the corresponding values of stage y and A are read from Fig. 5; (*f*) finally, Q is computed as $A\,V$. The relationship between Q and y is the desired rating curve, provided that all flow is confined to the

active channel. If overbank flow occurs, then an estimated allowance for overbank flow must be made. In the last column of Table 2, Manning's n values, as computed, are shown as a matter of interest. These, of course, are not required for determination of the relationship between Q and y by the

TABLE 2[a].—Typical Computations to Determine Relationship Between Stage and Flow

R' (1)	$(R')^{\frac{1}{2}}$ (1a)	V Eq. 18 (2a)	ψ' Eq. 19 (2b)	V/V''_f Fig. 3 (3)	V''_f (4a)	$R''=(V''_f)^2/gS$ (4b)	$R=R'+R''$ (5a)	Stage y Fig. 5 (5b)	A Fig. 5 (5c)	$Q=AV$ (6)	Manning's n Eq. 3 (7)
1.00	1.000	2.83	4.40	13.3	0.213	2.81	3.81	6.7	1,010	2,860	0.029
2.00	1.587	4.49	2.20	20.4	0.220	3.00	5.00	8.1	1,380	6,200	0.022
3.00	2.080	5.89	1.47	27.5	0.214	2.85	5.85	9.0	1,640	9,630	0.018

[a] All dimensions in the foot-pound-second system.

method described. The obvious advantage of this method is that the results do not depend upon individual judgment for selection of the appropriate n value.

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